2.3. Solved Problems – Gear Motion and Spur Gear Forces

Problem 1

A pinion gear with 22 teeth and a module of 6 mm has a rotational speed of 1200 rpm and drives a gear at 660 rpm. Determine:

- ***i) The number of teeth on the gear, and;***
- ***ii) The theoretical centre distance***

Solution

Data

z_p = 22 teeth
m = 6 mm
N_p = 1200 rpm
N_g = 660 rpm
z_g = ? teeth
a = ?mm

i) $$\frac{N_p}{N_g} = \frac{z_g}{z_p}$$

$$\Rightarrow \frac{1200}{660} = \frac{z_g}{22}$$

$$\Rightarrow z_g = 22 \times \frac{1200}{660}$$

$$\Rightarrow z_g = 40$$

Therefore there are 40 teeth on the gear.

ii) Theoretical centre distance

$$a = r_g + r_p$$

$$= \frac{D_g}{2} + \frac{D_p}{2}$$

$$= \frac{mz_g}{2} + \frac{mz_p}{2}$$

$$= \frac{6 \times 40}{2} + \frac{6 \times 22}{2}$$

$$= 120 + 66$$

$$= 186mm$$

Therefore the theoretical centre distance for the pinion and the gear is 186 mm.

Problem 2

A pair of gears has been designed with a velocity ratio of 3.20. The pinion has 20 teeth and the circular pitch is 78.54 mm. Determine:

i) ***The number of teeth on the driven gear.***
ii) ***The module for the gears.***
iii) ***The theoretical centre distance.***

Solution

Data

i = 3.20

z_p = 20 teeth

p_c = 78.54 mm

z_g = ? teeth

m = ?mm

a = ?mm

i) For the velocity ratio, $i = \frac{N_p}{N_g} = \frac{z_g}{z_p}$

$$\Rightarrow 3.2 = \frac{z_g}{20}$$

$$\Rightarrow z_g = 3.2 \times 20 = 64$$

Therefore there are 64 teeth on the driven gear.

ii) For the circular pitch, $p_c = \frac{\pi D}{z}$

$$\Rightarrow 78.54 = \frac{\pi D_p}{20}$$

$$\Rightarrow D_p = \frac{78.54 \times 20}{\pi} = 500mm$$

To calculate the module, $m = \frac{D_p}{z_p}$

$$\Rightarrow m = \frac{500}{20}$$

$$\Rightarrow m = 25mm$$

iii) To calculate the theoretical centre distance;

$$a = r_g + r_p$$

$$= \frac{D_g}{2} + \frac{D_p}{2}$$

$$= \frac{mz_g}{2} + \frac{500}{2}$$

$$= \frac{25 \times 64}{2} + 250$$

$$= 800 + 250$$

$$= 1050mm$$

Therefore the theoretical centre distance is 1050 mm.

Problem 3

A gear drive consists of two gears, A and B, and has a velocity ratio of 1.50. Gear A, the smaller of the two gears, revolves at 126 rpm in the clockwise direction, and has 28 teeth. If the gears have a module of 2 mm, determine:

- ***i) The number of teeth on Gear B.***
- ***ii) The pitch (reference) diameters for the two gears.***
- ***iii) The addendum.***
- ***iv) The dedendum.***
- ***v) The circular pitch.***
- ***vi) The tooth thickness.***
- ***vii) The speed of Gear B.***
- ***viii) The theoretical centre distance of the two gears.***

Solution

Data

i = 1.50

z_A = 28 teeth

N_A = 126 rpm

m = 2 mm

i) For the velocity ratio, $i = \frac{N_A}{N_B} = \frac{z_B}{z_A}$

$$\Rightarrow 1.5 = \frac{z_B}{28}$$

$$\Rightarrow z_B = 1.5 \times 28 = 42$$

Therefore there are 42 teeth on Gear B.

ii) For the pitch diameters of the two gears;

$$D_A = m \times z_A \qquad D_B = m \times z_B$$
$$= 2 \times 28 \qquad = 2 \times 42$$
$$= 56mm \qquad = 84mm$$

iii) For the addendum, refer to Table 1.1, therefore: Addendum = m = 2 mm

iv) For the dedendum, refer to Table 1.1, therefore: Dedendum = 1.25 x m = 1.25 x 2 = 2.5 mm

v) For the circular pitch, $p_c = \frac{\pi D_A}{z_A}$ or $p_c = \frac{\pi D_B}{z_B}$

$$\Rightarrow p_c = \frac{\pi 56}{28} \quad \text{or} \quad \Rightarrow p_c = \frac{\pi 84}{42}$$

$$\Rightarrow p_c = 2\pi = 6.28mm \quad \Rightarrow p_c = 2\pi = 6.28mm$$

vi) For the tooth thickness, refer to Table 2.1, therefore: t = m/2 = 2/2 = 1 mm

vii) $\frac{N_A}{N_B} = \frac{z_B}{z_A}$

$$\Rightarrow N_B = N_A \times \frac{z_A}{z_B}$$

$$\Rightarrow N_B = 126 \times \frac{28}{42}$$

$$\Rightarrow N_B = 84 rpm$$

Therefore the speed of gear B is 84 rpm in the anticlockwise direction.

viii) To calculate the theoretical centre distance;

$$a = r_B + r_A$$

$$= \frac{D_B}{2} + \frac{D_A}{2}$$

$$= \frac{84}{2} + \frac{56}{2}$$

$$= 42 + 28$$

$$= 70 mm$$

Therefore the theoretical centre distance is 70 mm.

Problem 4

The set of double-reduction gears shown in Figure 2.2, is driven by a pinion (Gear A) with a module of 1.5 mm, which rotates at 3600 rpm and has a pitch-line velocity of 4.52 ms^{-1}. The second set of gears (Gears C and D) has a pitch-line velocity of 0.78 ms^{-1} and a module of 2.5 mm. Given that the reduction ratio between the Gears A and B is to be 12:1, and the reduction ratio between Gears C and D is to be 10:1, determine:

- ***i) The number of teeth on each of the gears.***
- ***ii) The speed of Gears B, C, and D.***
- ***iii) The centre distances for Gears A and B, and Gears C and D.***

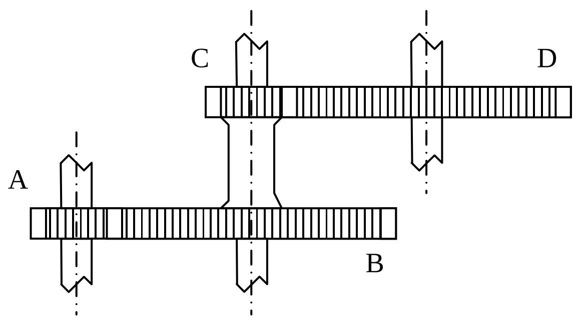

Figure 2.2 Spur Gear arrangement for Problem 4

Solution

Data

m_{AB} = 1.5 mm

N_A = 3600 rpm

$v_A = v_B$ = 4.52 ms^{-1}

i_{AB} = 12

i_{CD} = 10

$v_C = v_D = 0.78\ ms^{-1}$
$m_{CD} = 2.5$ mm

a. The pitch velocity is given by, $v_A = \frac{\pi D_A N_A}{60}$

$$\Rightarrow D_A = \frac{4.52 \times 60}{\pi \times 3600}$$
$$\Rightarrow D_A = 0.024m$$
$$= 24mm$$

Therefore the number of teeth on Gear A is given by, $d_A = mz_A$

$$\Rightarrow z_A = \frac{D_A}{m}$$
$$\Rightarrow z_A = \frac{24}{1.5}$$
$$= 16$$

Therefore there are 16 teeth on Gear A.

To calculate the number of teeth on Gear B, $i = \frac{z_B}{z_A}$

$$\Rightarrow 12 = \frac{z_B}{16}$$
$$\Rightarrow z_B = 12 \times 16 = 192$$

Therefore there are 192 teeth on Gear B.

To calculate the number of teeth on Gear C, the pitch velocity is given by, $v_C = \frac{\pi D_C N_C}{60}$

Where; $N_C = N_B = N_A \times \frac{z_A}{z_B}$

$$\Rightarrow N_C = 3600 \times \frac{16}{192}$$
$$\Rightarrow N_C = 300rpm$$

Therefore:

$$\Rightarrow D_C = \frac{0.78 \times 60}{\pi \times 300}$$
$\Rightarrow D_A = 0.050m$ (rounded to the nearest whole millimeter)
$$= 50mm$$

Therefore the number of teeth on Gear C is given by, $d_C = mz_C$

$$\Rightarrow z_C = \frac{D_C}{m}$$

$$\Rightarrow z_C = \frac{50}{2.5}$$

$$= 20$$

Therefore there are 20 teeth on Gear C.

To calculate the number of teeth on Gear D, $i = \frac{z_D}{z_C}$

$$\Rightarrow 10 = \frac{z_D}{20}$$

$$\Rightarrow z_D = 10 \times 20 = 200$$

Therefore there are 200 teeth on Gear D.

b. To calculate the speeds of the remaining gears, $N_B = N_C = 300rpm$, and;

$$N_D = N_C \times \frac{z_C}{z_D}$$

$$= 300 \times \frac{20}{200}$$

$$= 30rpm$$

c. To calculate the theoretical centre distances;

$$a_{AB} = r_B + r_A$$

$$= \frac{D_B}{2} + \frac{D_A}{2}$$

$$= \frac{1.5 \times 192}{2} + \frac{24}{2}$$

$$= 156mm$$

And

$$a_{CD} = r_C + r_D$$

$$= \frac{D_C}{2} + \frac{D_D}{2}$$

$$= \frac{50}{2} + \frac{2.5 \times 200}{2}$$

$$= 275mm$$

Problem 5

A spur pinion gear of 120 mm diameter is required to transmit a power of 5 kW to a spur gear, with both gears having a pressure angle of 20^0. Given that the pinion rotates in the anticlockwise direction at 200 rpm, and that the velocity ratio for the two gears is 2.50, determine:

- ***i) The diameter and rotational speed of the driven gear.***
- ***ii) The forces on each of the gears, and show on a free body diagram.***

Solution

Data

D_p = 120 mm
N_p = 200 rpm
P = 5 kW = 5 x 10^3 W
i = 2.5
$\phi = 20^0$

i) For the velocity ratio, $i = \frac{N_p}{N_g}$

$$\Rightarrow 2.5 = \frac{200}{N_g}$$

$$\Rightarrow N_g = \frac{200}{2.5} = 80 rpm$$

Therefore the gear rotates at 80 rpm in the clockwise direction.

Also: $\frac{N_p}{N_g} = \frac{z_g}{z_p} = \frac{D_g}{D_p}$

$$\Rightarrow \frac{D_g}{120} = \frac{200}{80}$$

$$\Rightarrow D_g = \frac{200}{80} \times 125 = 300 mm$$

ii) For the forces, the torque needs to be calculated first, $T = \frac{60 \times P}{2\pi N}$

$$\Rightarrow T = \frac{60 \times 5000}{2\pi 200}$$

$$= 238.78 Nm$$

Therefore, the forces on the pinion:

$$F_t = \frac{T}{r}$$

$$= \frac{238.78}{(0.12/2)}$$

$$= 3979.62 N$$

$$\begin{aligned}F_r &= F_t \tan\phi \\ &= 3979.62 \tan 20 \\ &= 1448.46N\end{aligned}$$

As confirmation, the forces on the gear can be calculated separately. Firstly;

$$\Rightarrow T = \frac{60 \times 5000}{2\pi 80}$$
$$= 596.83Nm$$

Therefore, the forces on the gear:

$$\begin{aligned}F_t &= \frac{T}{r} \\ &= \frac{596.83}{(0.30/2)} \\ &= 3978.87N \\ &\approx 3979.62N\end{aligned}$$

$$\begin{aligned}F_r &= F_t \tan\phi \\ &= 3978.87 \tan 20 \\ &= 1448.19N \\ &\approx 1448.46N\end{aligned}$$

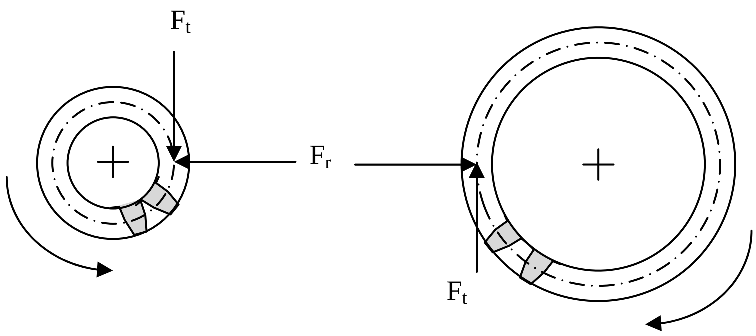

Figure 2.3 Spur Gear force directions for Problem 5

Problem 6

The spur gear arrangement shown in Figure 2.4 has a pinion (driver Gear A) that rotates at 625 rpm, an idler Gear B and a driven gear (Gear C). The gears have a module of 5 mm and a pressure angle of 20^0. Given the number of teeth as shown in Figure 2.4, determine:

i) The pitch diameters of each of the gears.
ii) The torque that each shaft is required to transmit.
iii) The forces on each of the gears, shown on a free body diagram.
iv) The resultant force on the shaft carrying Gear B.

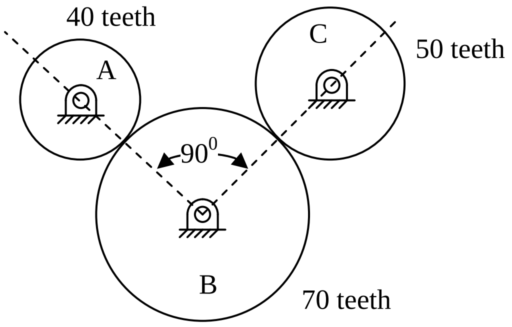

Figure 2.4 Spur Gear arrangement for Problem 6

Solution

Data

P = 3.2 kW = 3.2 x 10^3 W

N_A = 625 rpm

z_A = 40 teeth

z_B = 70 teeth

z_C = 50 teeth

m = 5 mm

$\phi = 20^0$

i) For the pitch diameters of each of the gears;

$$D_A = m \times z_A = 5 \times 40 = 200mm \qquad D_B = m \times z_B = 5 \times 70 = 350mm \qquad D_C = m \times z_C = 5 \times 50 = 250mm$$

ii) For the torque on each shaft, the first step is to know the speed of each gear, based on the velocity ratio, $i = \frac{N_A}{N_B} = \frac{z_B}{z_A}$;

$$\Rightarrow N_B = N_A \times \frac{z_A}{z_B}$$

$$\Rightarrow N_B = 625 \times \frac{40}{70}$$

$$\Rightarrow N_B = 357.14rpm$$

And

$$\Rightarrow N_C = N_B \times \frac{z_B}{z_C}$$

$$\Rightarrow N_C = 357.14 \times \frac{70}{50}$$
$$\Rightarrow N_C = 500rpm$$

Finally, as Gear B is an idler, it can also be shown that:

$$\Rightarrow N_C = N_A \times \frac{z_A}{z_C}$$
$$\Rightarrow N_C = 625 \times \frac{40}{50}$$
$$\Rightarrow N_C = 500rpm$$

That is, the idler gear has no effect on the final speed of the gear train, however it does change the rotation direction.
Now that each of the rotational speeds is known, the torque can be calculated from:

$$T_A = \frac{60 \times P}{2\pi N_A} = \frac{60 \times 3200}{2\pi 625} = 48.89Nm$$

$$T_B = 0$$ (idler gear)

$$T_C = \frac{60 \times P}{2\pi N_C} = \frac{60 \times 3200}{2\pi 500} = 61.12Nm$$

iii) Therefore, the forces on Gear A:

$$F_{tA} = \frac{T}{r_A} = \frac{48.89}{(0.2/2)} = 488.92N$$

$$F_{rA} = F_{tA} \tan\phi = 488.92 \tan 20 = 177.95N$$

And the forces on Gear C:

$$F_{tC} = \frac{T}{r_C} = \frac{61.12}{(0.25/2)} = 488.92N = F_{tA}$$

$$F_{rC} = F_{tC}\tan\phi$$
$$= 488.92\tan 20$$
$$= 177.95N$$
$$= F_{rA}$$

The free body diagram is shown in Figure 2.5

iv) The resultant force on the shaft carrying Gear B;

$$F_B = \sqrt{2(F_{tA}+F_{rA})^2}$$
$$= \sqrt{2(488.92+177.95)^2}$$
$$= 943.10N$$

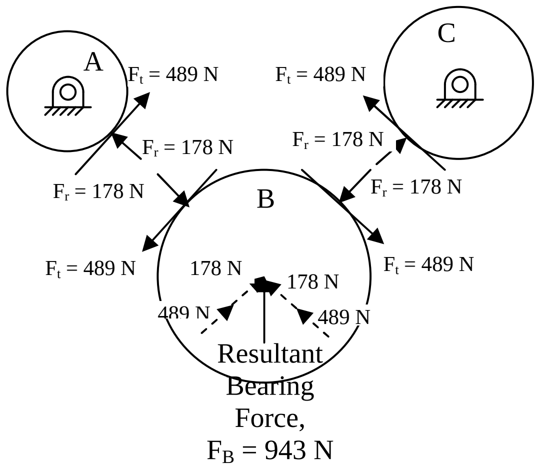

Figure 2.5 Spur Gear force directions for Problem 6

Problem 7
Based on the gear arrangement shown in Figure 2.2, and for an input power of 4.25 kW, determine the forces on each of the gears assuming that the pressure angle is 20^0, and show these on a free body diagram.

Solution

Continued from Problem 4 with the following Data

$P = 4.25$ kW $= 4.25 \times 10^3$ W
$N_A = 3600$ rpm
$N_B = N_C = 300$ rpm
$N_D = 30$ rpm

$D_A = 24$ mm
$D_B = 288$ mm
$D_C = 50$ mm
$D_D = 500$ mm

$m = 5$ mm
$\phi = 20^0$

Starting with Gear A;

$$T_A = \frac{60 \times P}{2\pi N_A}$$
$$= \frac{60 \times 4250}{2\pi 3600}$$
$$= 11.27 Nm$$

$$F_{tA} = \frac{T}{r_A}$$
$$= \frac{11.27}{(0.024/2)}$$
$$= 939.46 N$$

$$F_{rA} = F_{tA} \tan\phi$$
$$= 939.46 \tan 20$$
$$= 341.93 N$$

The forces on Gear B will be equal and in opposite directions to these forces, so the next calculation is for Gear C.

$$T_C = \frac{60 \times P}{2\pi N_C}$$
$$= \frac{60 \times 4250}{2\pi 300}$$
$$= 135.28 Nm$$

$$F_{tC} = \frac{T}{r_C}$$
$$= \frac{135.28}{(0.050/2)}$$
$$= 5411.27 N$$

$$F_{rC} = F_{tC} \tan\phi$$
$$= 5411.27 \tan 20$$
$$= 1969.54 N$$

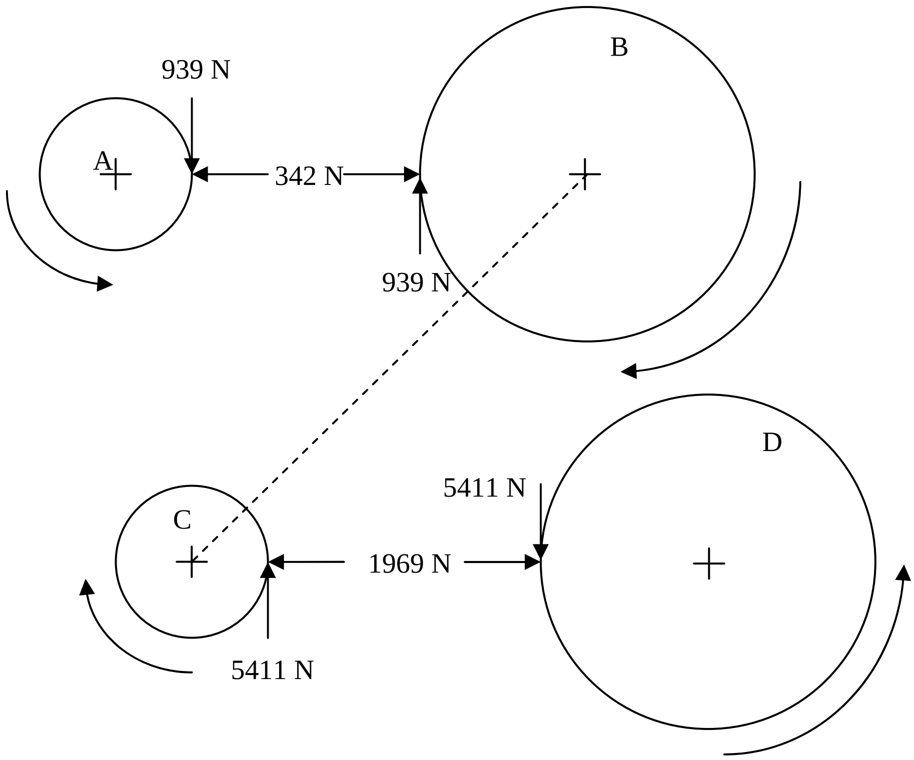

Figure 2.6 Spur Gear force directions for Problem 7